VCU Discrete Mathematics Seminar

Equitable Colorings of Cubic Graphs & Stemock's conjecture

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Wednesday, Sept. 6
1:00-1:50 EST

In person! in 4145 Harris Hall, and Zoom @
`https://vcu.zoom.us/j/92975799914`
`password=graphs2357`

A k-total coloring of a simple graph $G = (V, E)$ assigns at most k colors to the vertices and edges of G such that distinct colors are assigned to every pair of adjacent vertices in V, to every pair of adjacent edges in E, and each vertex and its incident edges. A total coloring is equitable if the cardinalities of any two color classes differ by at most 1.

In 2020, Stemock considered equitable total colorings of cubic graphs in his paper "On the equitable total $(k{+}1)$-coloring of k-regular graphs". The author conjectured that every 4-total coloring of a cubic graph is equitable if $n < 20$. This conjecture becomes relevant when we realize that it refers to more than 40000 graphs, which can be verified on the website "House of Graphs".

We found cubic graphs that serve as counterexamples to Stemock's conjecture and display them in the text. Then, we studied the cubic graphs for $n < 20$ and determined that a 4-total coloring is necessarily equitable on cubic graphs of 6, 8, 10, and 14 vertices. Furthermore, we prove that a cubic graph of 12 vertices is the smallest possible counterexample to Stemock's conjecture and that 14 is the largest value of n which implies that all 4-total colorings are equitable.

For the DM seminar schedule, see:

`https://go.vcu.edu/discrete`