## VCU Discrete Mathematics Seminar

## On the existence of ( $\mathrm{r}, \mathrm{g}, \mathrm{\chi}$ )-graphs

## Prof Linda Lesniak (Western Michigan University)

Wednesday, Mar. 13
1:00-1:50 EST
In person in 4145 Harris Hall, and on Zoom @
https://vcu.zoom.us/j/92975799914 password=graphs2357


In 1961, Erdős showed the following: Given integers $\chi \geqslant 3$ and $g \geqslant 3$, there exists a graph with chromatic number $\chi$ and girth g . In 1947, Tutte asked the following question : Given integers $r \geqslant 2$ and $g \geqslant 3$, does there exist an r-regular graph with girth $g$ ? Erdős and Sachs established existence for all pairs $r, g$ in 1960. Finally, Rubio-Montiel considered the problem of the existence of graphs with a given regularity $r$ and chromatic number $\chi$.

In this talk we'll look at the question of the existence of $(r, g, \chi)$-graphs, that is, r-regular graphs of girth $g$ with chromatic number $\chi$. These graphs were introduced by Gabriela Araujo-Pardo, Zhanar Berikkyzy and Linda Lesniak, where the emphasis was on the case $\chi=3$. But we now know, for example, according to work of Gabriela Araujo-Pardo, Julio Diaz-Calderon, Julian Fresan, Diego Gonzales-Moreno, $\mathrm{L}^{*} 2$ and Mika Olsen, that if g and $\chi$ are integers both at least 3 , then for $r$ sufficiently large there exist ( $r, g, \chi$ )-graphs.

Finally, we'll consider the question of the existence of "equitable" (r, g, $\chi$ )graphs, that is, $(r, g, \chi)$-graphs with a $\chi$-coloring in which the color classes differ in size by at most 1 .

